647510 **BSc3MatC0301** **Seat No :________**

B.Sc. Semester - 3 (*CBCS*) Examination

December. -2020 [OLD COURSE]

MATHEMATICS (CORE)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figure to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Que-1

(A) ATTEMPT ANY ONE : **07**

(1) Define: Convergence of a sequence.
Prove that the limit of a sequence is unique if the sequence is convergent.

(2) Prove that a sequence $\{S_n\}$ is convergent
$\boldsymbol{iff}\ \forall \boldsymbol{\epsilon} > \mathbf{0}, \exists \boldsymbol{m} \in \mathbb{N}\ \ni \left|\boldsymbol{S_{n+p}} - \boldsymbol{S_n}\right| < \boldsymbol{\epsilon}, \forall \boldsymbol{n} \geq \boldsymbol{m}\ \&\ \boldsymbol{p} \geq \mathbf{1}$

(B) ATTEMPT ANY ONE: **04**

(1) For the sequences $\{a_n\}$ & $\{b_n\}$, if $\lim\limits_{n\to\infty} a_n = a$ & $\lim\limits_{n\to\infty} b_n = b$ then prove that $\lim\limits_{n\to\infty}(a_n \pm b_n) = \lim\limits_{n\to\infty} a_n \pm \lim\limits_{n\to\infty} b_n = a \pm b$.

(2) Discuss convergence and find limit of the sequence $\{S_n\}$ if exists where $S_1 = \sqrt{2}$, $S_{n+1} = \sqrt{2S_n}$, where $n \in \mathbb{N}$.

(C) ATTEMPT ANY ONE : **03**

(1) Show that $\lim\limits_{n\to\infty}\{\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots \frac{1}{\sqrt{n^2+n}}\} = 1.$

(2) Show that $\lim\limits_{n\to\infty}\frac{1}{n}\ [1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}] = 0.$

Que-2

(A) ATTEMPT ANY ONE : **07**

(1) Define: Series, Geometric Series
Test the convergence of the series $\mathbf{1} + \frac{\mathbf{1}}{\mathbf{2}} + \frac{\mathbf{1}}{\mathbf{4}} + \frac{\mathbf{1}}{\mathbf{8}} + \cdots \infty.$

(2) Discuss Comparision Tests.
Test the convergence of the series $\frac{\mathbf{1}}{\mathbf{1\cdot2\cdot3}} + \frac{\mathbf{3}}{\mathbf{2\cdot3\cdot4}} + \frac{\mathbf{5}}{\mathbf{3\cdot4\cdot5}} +$

(B) ATTEMPT ANY ONE: **04**

(1) Explain: D'Alembert's Ratio Test.
Test the convergence of the series $\sum_{\boldsymbol{n}=\mathbf{1}}^{\infty}\frac{\boldsymbol{n}^{\mathbf{2}}}{\mathbf{3}^{\boldsymbol{n}}}$.

(2) Explain: Rabbe's Test
Test the convergence of the series $\sum_{\boldsymbol{n}=\mathbf{1}}^{\infty}\frac{\mathbf{3\cdot6\cdot9\cdot\ldots\cdot(3}\boldsymbol{n}\mathbf{)}}{\mathbf{7\cdot10\cdot13\cdot\ldots(3}\boldsymbol{n}\mathbf{+4)}}\cdot \boldsymbol{x}^{\boldsymbol{n}}, \boldsymbol{x} > \mathbf{0}.$

(C) ATTEMPT ANY ONE : **03**

(1) State Leibnitz's test for alternating series.

(2) Test the convergence of the series $\sum_{\boldsymbol{n}=\mathbf{1}}^{\infty}\frac{\sqrt{\boldsymbol{n}}}{(\boldsymbol{n}+\mathbf{1})^{\mathbf{2}}}$.

Que-3

(A) ATTEMPT ANY ONE : **07**

(1) Define: Gradient
If ϕ and ψ are differentiable scalar functions, then prove the followings:
(i)$\boldsymbol{\nabla}(\boldsymbol{\phi\psi}) = \boldsymbol{\phi}\boldsymbol{\nabla}(\boldsymbol{\psi}) + \boldsymbol{\psi}\boldsymbol{\nabla}(\boldsymbol{\phi})$ (ii) $\boldsymbol{\nabla}\left(\frac{\boldsymbol{\phi}}{\boldsymbol{\psi}}\right) = \frac{\boldsymbol{\psi}\boldsymbol{\nabla}(\boldsymbol{\phi}) - \boldsymbol{\phi}\boldsymbol{\nabla}(\boldsymbol{\psi})}{\boldsymbol{\psi}^{\mathbf{2}}}$

(2) Define: Laplacian Operator
Prove that, $\boldsymbol{\nabla}^{\mathbf{2}}\boldsymbol{f}(\boldsymbol{r}) = \boldsymbol{f}''(\boldsymbol{r}) + \frac{\mathbf{2}}{\boldsymbol{r}}\boldsymbol{f}'(\boldsymbol{r})$ where $\boldsymbol{r} = |\bar{\boldsymbol{r}}|, \bar{\boldsymbol{r}} = (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}).$

(B)	**ATTEMPT ANY ONE :**	**04**
(1)	If $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \boldsymbol{x^2 y} + \boldsymbol{xy^2} + \boldsymbol{z^2}$, then find $\boldsymbol{\nabla\phi}$ and find unit normal at $(\mathbf{1}, \mathbf{2}, \mathbf{3})$**.**	
(2)	Show that the function $\boldsymbol{H} = \boldsymbol{e}^{-\lambda x}(\boldsymbol{c_1} \sin(\boldsymbol{\lambda y}) + \boldsymbol{c_2} \cos(\boldsymbol{\lambda y}))$ satisfies the Laplace equation where $\boldsymbol{\lambda}, \boldsymbol{c_1}, \boldsymbol{c_2}$ are constants.	
(C)	**ATTEMPT ANY ONE :**	**03**
(1)	Define: Solenoidal Function If $\boldsymbol{F} = (x^2z - axyz,\ xy - 3x^2yz,\ yz^2 - xz)$ is solenoidal, then find $\boldsymbol{a}$.	
(2)	Define: Irrotational Function. Show that $\boldsymbol{F} = (\boldsymbol{x^3}, \boldsymbol{y^3}, \boldsymbol{z^3})$ is irrotational.	

Que-4

(A)	**ATTEMPT ANY ONE :**	**07**
(1)	Define: Double Integration Explain evaluation of double integrals and also explain change of order of integration.	
(2)	Evaluate: $\iiint \left(\boldsymbol{a^2b^2c^2} - \boldsymbol{b^2c^2x^2} - \boldsymbol{c^2a^2y^2} - \boldsymbol{a^2b^2z^2}\right)\boldsymbol{dxdydz}$ over R, where R is ellipsoid $\frac{\boldsymbol{x^2}}{\boldsymbol{a^2}} + \frac{\boldsymbol{y^2}}{\boldsymbol{b^2}} + \frac{\boldsymbol{z^2}}{\boldsymbol{c^2}} = \mathbf{1}$.	
(B)	**ATTEMPT ANY ONE:**	**04**
(1)	Evaluate: $\iint \boldsymbol{xydydx}$ over R , whrere R is the first quadrant of the circle $\boldsymbol{x^2} + \boldsymbol{y^2} = \mathbf{9}$.	
(2)	Evaluate: $\int_{\mathbf{0}}^{\boldsymbol{a}}\int_{\mathbf{0}}^{\boldsymbol{b}}\int_{\mathbf{0}}^{\boldsymbol{c}}(\boldsymbol{x} + \boldsymbol{y} + \boldsymbol{z})\boldsymbol{dxdydz}$.	
(C)	**ATTEMPT ANY ONE:**	**03**
(1)	Find the area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by using double integration.	
(2)	Evaluate: $\int_{\mathbf{0}}^{\mathbf{1}}\int_{\mathbf{0}}^{\mathbf{1}-\boldsymbol{x}}\int_{\mathbf{0}}^{\boldsymbol{x}+\boldsymbol{y}} \boldsymbol{e^z dzdydx}$.	

Que-5

(A)	**ATTEMPT ANY ONE:**	**07**
(1)	State Gauss divergence theorem and verify the theorem for $\boldsymbol{F} = (4x, -2y^2, z^2)$ where R is $\boldsymbol{x^2} + \boldsymbol{y^2} = \mathbf{4}, \boldsymbol{z} = \mathbf{0}$ & $\boldsymbol{z} = \mathbf{3}$.	
(2)	Prove that $\boldsymbol{\beta}(\boldsymbol{p}, \boldsymbol{q}) = \frac{\boldsymbol{\Gamma p \Gamma q}}{\boldsymbol{\Gamma(p+q)}}$, $\boldsymbol{where\ p} > \mathbf{0}$ & $\boldsymbol{q} > \mathbf{0}$.	
(B)	**ATTEMPT ANY ONE :**	**04**
(1)	Verify the Stoke's theorem for $\boldsymbol{F} = (\mathbf{2}\boldsymbol{x} - \boldsymbol{y}, -\boldsymbol{yz^2}, -\boldsymbol{y^2z})$ & S is the upper half of the unit sphere.	
(2)	Find the value of $\boldsymbol{\Gamma}\frac{\mathbf{1}}{\mathbf{4}}\ \boldsymbol{\Gamma}\frac{\mathbf{3}}{\mathbf{4}}$.	
(C)	**ATTEMPT ANY ONE :**	**03**
(1)	Prove: $\boldsymbol{\beta}(\boldsymbol{m}, \boldsymbol{n}) = \boldsymbol{\beta}(\boldsymbol{m} + \mathbf{1}, \boldsymbol{n}) + \boldsymbol{\beta}(\boldsymbol{m}, \boldsymbol{n} + \mathbf{1})$.	
(2)	Prove: $\int_{\mathbf{0}}^{\infty} \frac{\mathbf{x^{a-1}}}{\mathbf{(1+x)}}\mathbf{dx} = \mathbf{\Gamma a\, \Gamma(1 - a)}$.	

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